Computational Physics

Dry Solid Friction: Force and Torque, Static and Dynamic

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Motivation

Static friction:

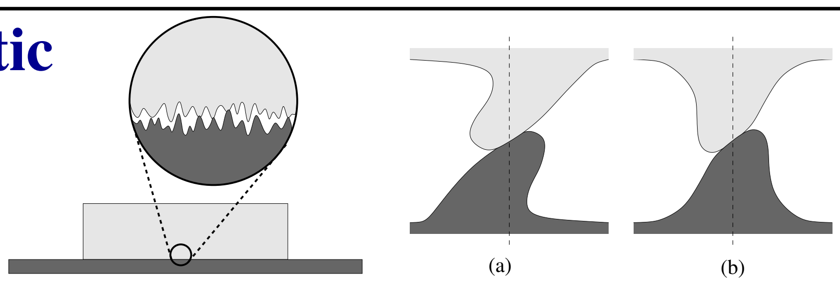
The difference in static friction and dynamic friction is observed in many experiments. Its origin has been intensively studied for a long time (starting probably with Euler 1750) and explained with several concepts.

Here it shall be explained with a simple mechanism involving tangential springs, modelling interlocked asperities between two contacting surfaces. The key concept is *strain coherence* of these interlocked asperities which leads to a difference in static and dynamic friction.

Dynamic friction:

Due to multiple degrees of freedom the corresponding different dynamic friction processes are principally coupled: we focus here on the coupling between sliding and spinning motion.

Static



Left: Solid bodies in contact: microscopic roughness is responsible for dry friction. *Right*: Plastic creep reduce elastic tangential strain at interlocked asperities. A freshly strained contact (a) quickly deforms plastically (b) if the tangential stress exceeds the yield treshold.

Ansatz friction force

$$f(s) = A\, n_{\mathrm{p}}\, k \int_0^\infty p(\ell)\, t(\ell, s)\, d\ell$$

with contact area A, number density n_p of of active pinning sites, spring constant k, probability distribution function $p(\ell)$ of asperities of size l, and the sawtooth shaped function $t(l,s) = s \bmod l$.

The phase of the l periodic function $t(l,s)$ is:

$$\phi(\ell, s) \equiv t(\ell, s)/\ell$$

The equation for $f(s)$ is closely related to the probability distribution of these phases:

$$w(\phi, s) = \int_0^\infty p(\ell)\, \delta\left(\phi - \frac{t(\ell, s)}{\ell}\right) d\ell.$$

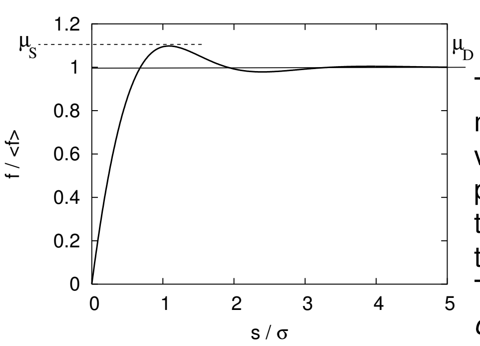
The initial states are coherent:

$$w(\phi, 0) = \delta(\phi)$$

After sufficient displacement s decoherence appears:

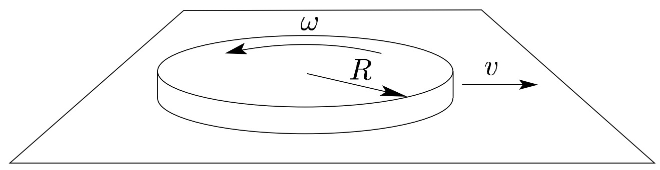
$$\lim_{s\to\infty} w(\phi, s) = 1.$$

This leads to a constant friction force $f(s)$ for large displacements.



The friction force as a function of the displacement s with $p(\ell) \propto \exp[-\ell^2/2\sigma^2]$. The maximum value of f is $\approx 1.098\langle f\rangle$ at $s \approx 1.087\sigma$. The peak represents the static friction force needed to set the body in motion, while the dynamic friction force needed to keep it in motion is smaller. This could be explained in terms of the concept of *coherence*.

Dynamic



The model is a sliding and spinning disk on a flat horizontal surface. The mass distribution of the disk is homogeneous.

Equations for the friction is:

$$\mathbf{F} = -\frac{\mu F_{\mathrm{n}}}{R^2\pi} \int_{\mathbf{r}\in A} \frac{\mathbf{v} + \boldsymbol{\omega}\times\mathbf{r}}{|\mathbf{v} + \boldsymbol{\omega}\times\mathbf{r}|} d^2r$$

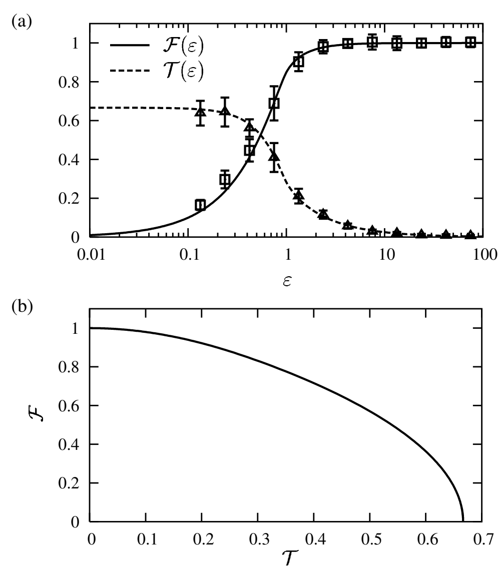
$$\mathbf{T} = -\frac{\mu F_{\mathrm{n}}}{R^2\pi} \int_{\mathbf{r}\in A} \mathbf{r} \times \frac{\mathbf{v} + \boldsymbol{\omega}\times\mathbf{r}}{|\mathbf{v} + \boldsymbol{\omega}\times\mathbf{r}|} d^2r$$

where R is the radius, μ the friction coefficient, $\mathbf{v}$ the velocity, $\boldsymbol{\omega}$ is the angular velocity of the disk. The integration extends over the area of the disk with $\mathbf{r}$ vectors starting at the center.

Analytic solution of the equations give complex functions with complete elliptic integrals. Their solutions are plotted below as the dimensionless forces

$$\mathcal{F}(\varepsilon) = \frac{F}{\mu_d F_n} \qquad \mathcal{T}(\varepsilon) = \frac{T}{\mu_d F_n R}$$

(a) The dimensionless friction force and torque (the analytic solutions of the above equations), $\mathcal{F}$ and $\mathcal{T}$, as functions of the dimensionless velocity parameter $\varepsilon = v/R\omega$. Squares and triangles with errorbars represent experimental data (explained in the text). (b) The friction force and torque are coupled: the curve shows the possible $(\mathcal{F}, \mathcal{T})$ pairs.



Conclusions

Model restricts ratio to the interval

$$1 \le \frac{\mu_{\mathrm{S}}}{\mu_{\mathrm{D}}} \le 2$$

and is a function with form:

$$\frac{\mu_{\mathrm{S}}}{\mu_{\mathrm{D}}} = g\left(\frac{\langle\ell^2\rangle - \langle\ell\rangle^2}{\langle\ell\rangle^2}\right).$$

From the solutions and further investigations one finds, that sliding and spinning motion stops at the same time and the ratio ϵ reaches a universal value $\epsilon \approx 0.653$.

Outlook

Investigation of the transition static –> dynamic